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A GENERALIZED STUDY OF PRECIPITATION FORECASTING

PART 3: COMPUTATION OF PRECIPITATION RESULTING FROM VERTICAL VELOCITIES DEDUCED FROM VORTICITY CHANGES

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ABSTRACT

Vertical velocities are deduced from two successive geopotential fields by means of a simplified vorticity equation. These velocities are used in a model described in earlier papers in this series to compute contemporary precipitation. Charts comparing computed with observed precipitation are shown.

INTRODUCTION

The computation of vertical velocity is an essential part of the problem of forecasting precipitation. In the preliminary exploratory phase of this investigation [1, 2], a precipitation model was developed relating the amount of precipitation to the fields of moisture and vertical velocity. Vertical velocities were computed by integrating with height the horizontal divergence of observed winds determined by a triangulation technique suggested by Bellamy [3]. Although the vertical velocities obtained by this kinematic method showed skill in delineating the areas over which precipitation occurred, the method is limited to a contemporary computation of precipitation since "true" winds are needed and forecasting these is at least as difficult as forecasting precipitation itself. In an attempt to obtain vertical velocities from a single geopotential field, various winds were derived—geostrophic and gradient winds using contour spacing and curvature, and gradient winds computed from a cubic surface fitted to reported heights; the resulting horizontal divergence and vertical velocities calculated from these winds were found to give insufficient information for computing precipitation. However, if one considers two successive geopotential fields, it is possible to deduce vertical velocities from changes in the vorticity of the geostrophic wind. The purpose of the present investigation was to study the usefulness of such vertical velocities in predicting precipitation.

COMPUTATION OF VERTICAL VELOCITY

The idea of using the vorticity equation to compute vertical motion is not a new one and has been advanced by several authors. The development used in this study is very similar to that given by Eliassen and Hubert [4] and consequently will be discussed only briefly here.

Using pressure as the vertical coordinate, the vorticity equation may be written

$$\frac{\partial \eta}{\partial t} = -\mathbf{V}\cdot\nabla\eta - \omega\frac{\partial \eta}{\partial p} + \eta\frac{\partial \omega}{\partial p} \tag{1}$$

where η denotes absolute vorticity (vertical component), $\mathbf{V}$ and ∇ are vectors in a constant pressure surface, p is pressure and ω is the individual rate of change of pressure $\left(=\frac{dp}{dt}\right)$. Frictional influences and the turning of the vortex lines are

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neglected. Neglect of the latter probably causes serious errors in small highly baroclinic areas, for example, near strong frontal regions. However, including these terms would have added considerably to the already enormous computations and they are therefore omitted.

Dividing through by η^2 and rearranging, equation (1) may be written

$$\frac{\partial}{\partial p}\left(\frac{\omega}{\eta}\right)=\frac{\frac{\partial \eta}{\partial t}+\mathbf{V}\cdot\nabla\eta}{\eta^2} \tag{2}$$

This can be integrated to obtain

$$\left(\frac{\omega}{\eta}\right)_{p=p_1}=\left(\frac{\omega}{\eta}\right)_{p=p_0}+\int_{p_0}^{p_1}\frac{\frac{\partial \eta}{\partial t}+\mathbf{V}\cdot\nabla\eta}{\eta^2}dp \tag{3}$$

Evaluating the integral by the trapezoidal rule gives

$$\left(\frac{\omega}{\eta}\right)_{p=p_1}=\left(\frac{\omega}{\eta}\right)_{p=p_0}+\frac{1}{2}\left[\left(\frac{\frac{\partial \eta}{\partial t}+\mathbf{V}\cdot\nabla\eta}{\eta^2}\right)_{p=p_1}+\left(\frac{\frac{\partial \eta}{\partial t}+\mathbf{V}\cdot\nabla\eta}{\eta^2}\right)_{p=p_0}\right](p_1-p_0) \tag{3a}$$

Thus, given the vorticity and its changes at various levels and ω at the lower boundary, we can obtain $\frac{\omega}{\eta}$, and thus ω, at any level. For instance, at 850 mb.,

$$\left(\frac{\omega}{\eta}\right)_{850}=\left(\frac{\omega}{\eta}\right)_{1000}-\frac{1}{2}\left[\left(\frac{\frac{\partial \eta}{\partial t}+\mathbf{V}\cdot\nabla\eta}{\eta^2}\right)_{1000}+\left(\frac{\frac{\partial \eta}{\partial t}+\mathbf{V}\cdot\nabla\eta}{\eta^2}\right)_{850}\right]150 \tag{3b}$$

Using the hydrostatic equation, an approximate relationship can be derived between ω and w, the vertical velocity, using height as a vertical coordinate.

$$w=\frac{dz}{dt}\approx-\frac{\omega}{\rho g} \tag{4}$$

where ρ is the density of air and g is the acceleration of gravity.

Vorticity was computed for four levels (1,000, 850, 700 and 500 mb.) by a graphical procedure described by Fjørtoft [5]. Briefly, the method is to shift the contour field a specified distance east and west and north and south and obtain a space-averaged contour field. Subtracting from this the original contour field gives a quantity proportional to the relative vorticity. Absolute vorticity, or more precisely a parameter roughly proportional to the absolute vorticity, is obtained by graphically adding in a term which is a function of latitude. The mesh length used was six degrees of latitude as suggested by Fjørtoft. Various other mesh lengths were tried but this one gave what was considered to be the best degree of smoothing.

Successive charts, 12 hours apart, were subtracted to obtain $\frac{\partial \eta}{\partial t}$. Since the quantity obtained was a 12-hour average rather than an instantaneous value, all other parameters were averaged over the 12-hour period. The advection of vorticity, $\mathbf{V}\cdot\nabla\eta$, was computed graphically and the values at the beginning and end of the 12-hour period were averaged.

These quantities were combined by graphical methods and integrated vertically as indicated by equation (3) to obtain vertical velocities at 850, 700, and 500 mb. These velocities were considered to be the mean for the 12-hour period. A typical set of isanabats for 0300–1500 GMT, January 3, 1953, is shown in figure 1B, C, D. The associated sea level chart for 0630 GMT is shown in figure 1A.

Two sets of computations were made, one by assuming that $\omega=0$ at 1,000 mb. which was assumed to be the surface pressure, and the other by introducing at the ground a topographically induced vertical motion. This was computed assuming

$$w_{1000}=\mathbf{V}\cdot\nabla h \tag{5}$$

where h is the height of the ground. Surface contours used were from a chart of smoothed broad-scale topography by Smagorinsky [6]. For the "advecting" wind, both observed "gradient level" winds and geostrophic winds at 850 mb. were used with substantially similar results. Topographic vertical velocities obtained at the beginning and end of a 12-hour period were averaged to give a value consistent with the average velocities deduced from equation (3). Figure 2 shows the vertical velocity fields computed for 0300–1500 GMT, January 3, 1953, taking into account the forced orographic ascent. This can be compared with figure 1, the results assuming a vertical velocity of zero at the surface. The flow was approximately from the northwest resulting in upward velocities on the western slopes of the Appalachians and downward velocities toward the east (fig. 2A). The effect of topography shows up most pronouncedly at the 850-mb. level where the isanabats are distorted north and southward along the mountain ridges.

COMPUTATION OF PRECIPITATION

Given the fields of vertical velocity and moisture, the amount of precipitation that will result in a 12-hour period was shown by Thompson and Collins [1] to be

$$P=\Sigma\{I[w-0.28(T-T_d)]\Delta z\} \tag{6}$$

where the atmosphere is divided into a number of layers and P, the 12-hour amount of precipitation, is the summation of the contributions of the various layers. For this present study three layers, approximately centered at 850, 700, and 500 mb. were used. In equation (6) I is a function of pressure and temperature, $(T-T_d)$ is the dewpoint depression in degrees Celsius and Δz is the thickness of the layer. As described by Kuhn [2], this equa-

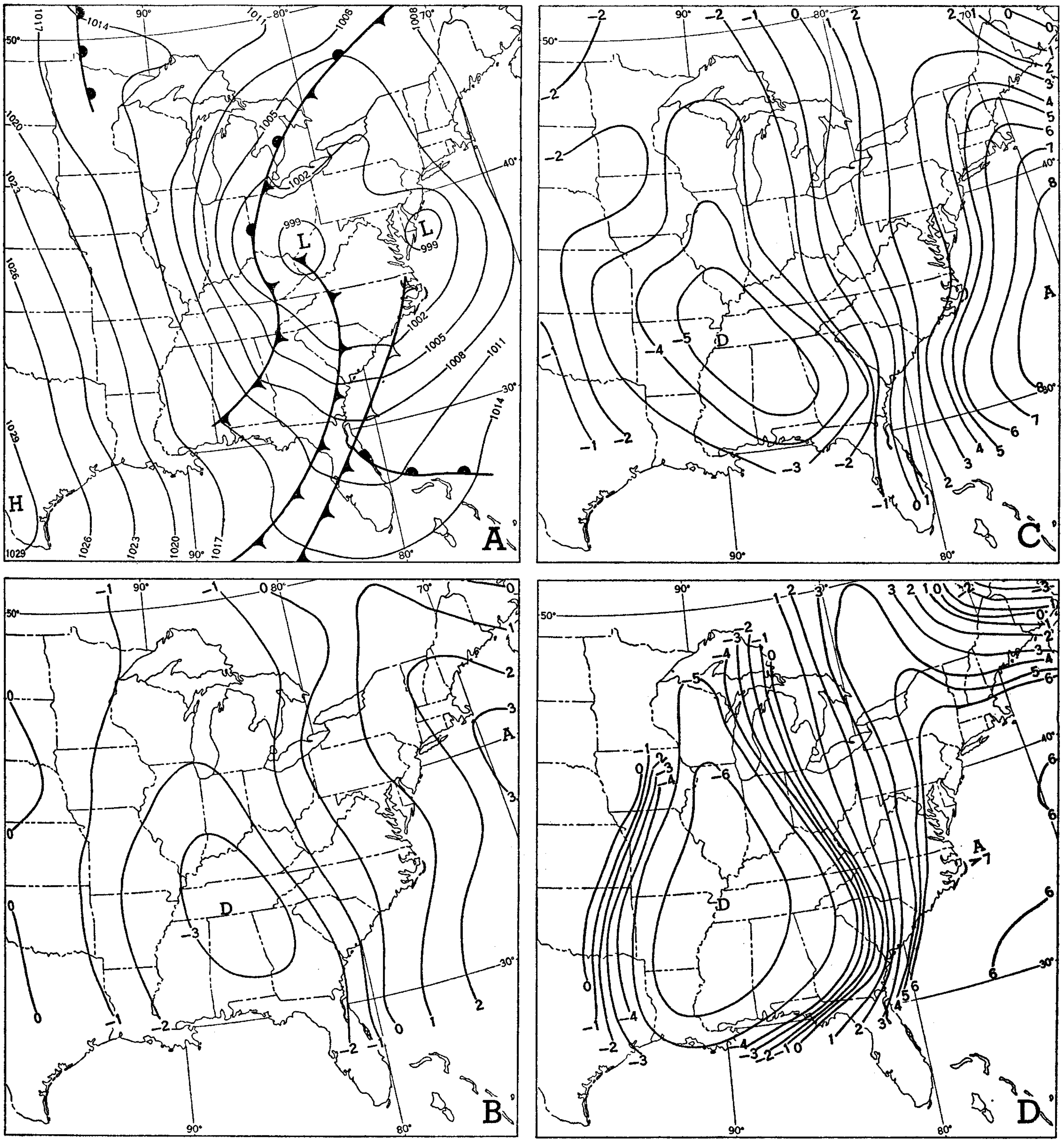

FIGURE 1.—(A) Surface chart, 0630 GMT, January 3, 1953. (B, C, D) Mean vertical velocity (cm. sec.$^{-1}$), 0300–1500 GMT, January 3, 1953, for 850, 700, and 500 mb., respectively. Positive values indicate ascent.

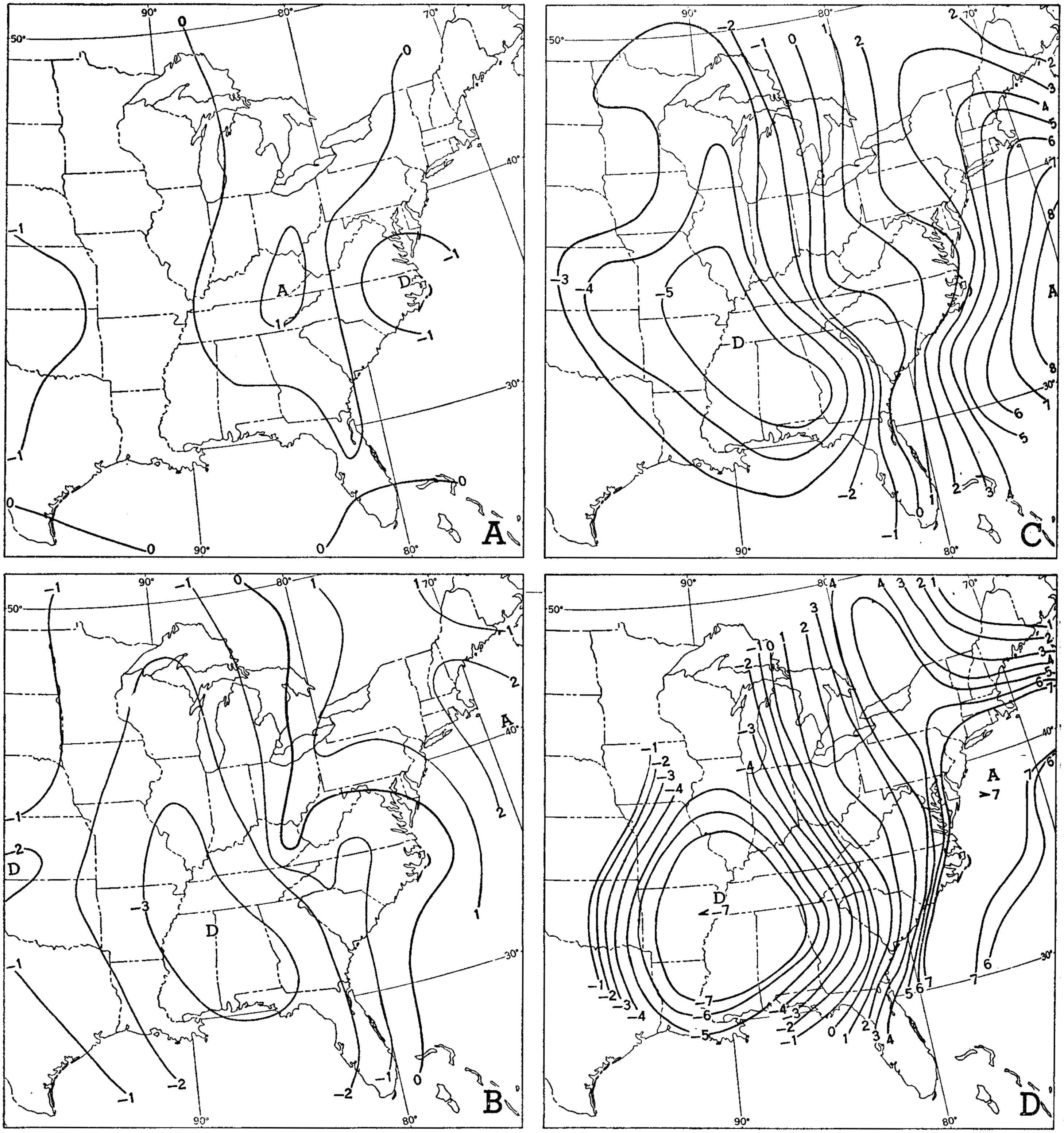

FIGURE 2.—Mean vertical velocity including topographic effects, 0300–1500 GMT, January 3, 1953 for (A) 1,000 mb., (B) 850 mb., (C) 700 mb., (D) 500 mb. Positive values indicate ascent.

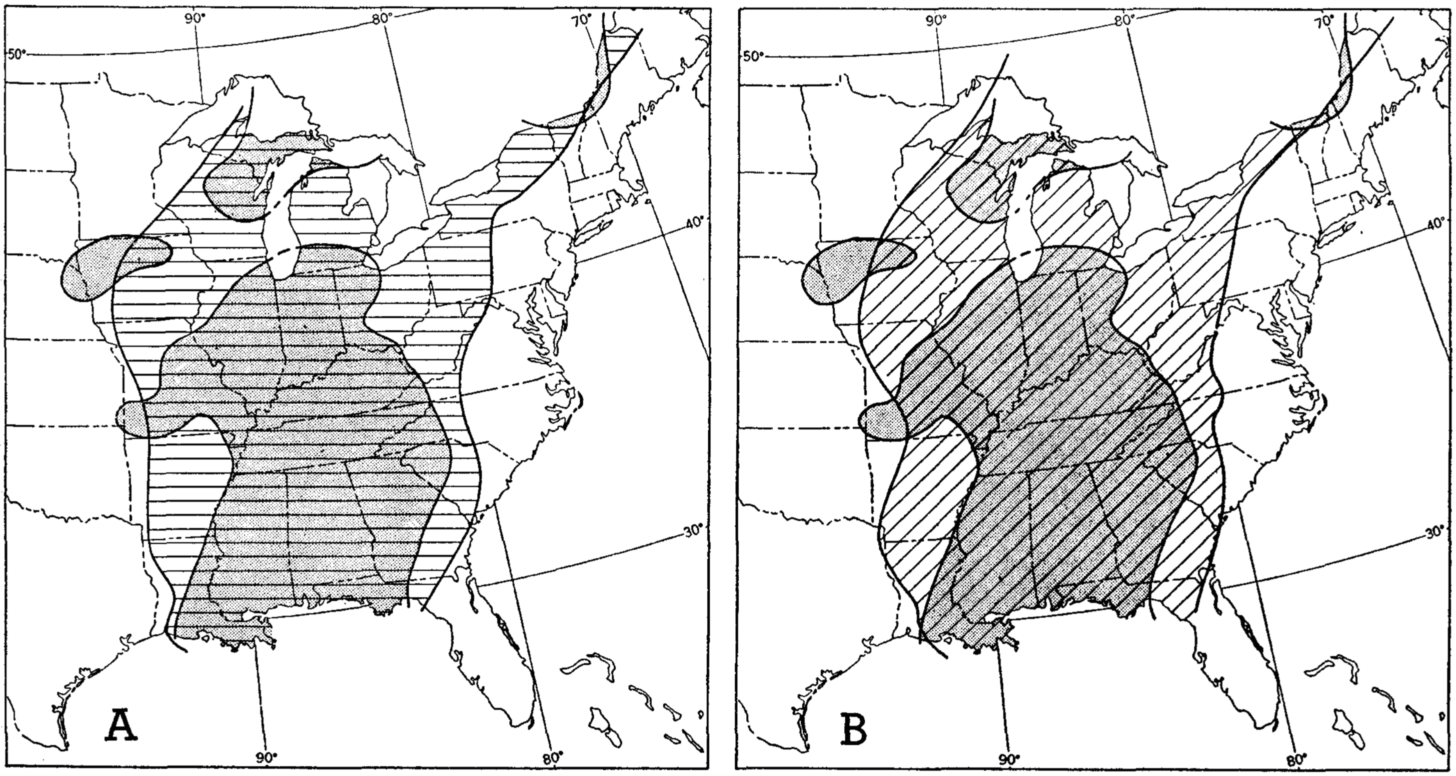

FIGURE 3.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 0300–1500 GMT, January 2, 1953.

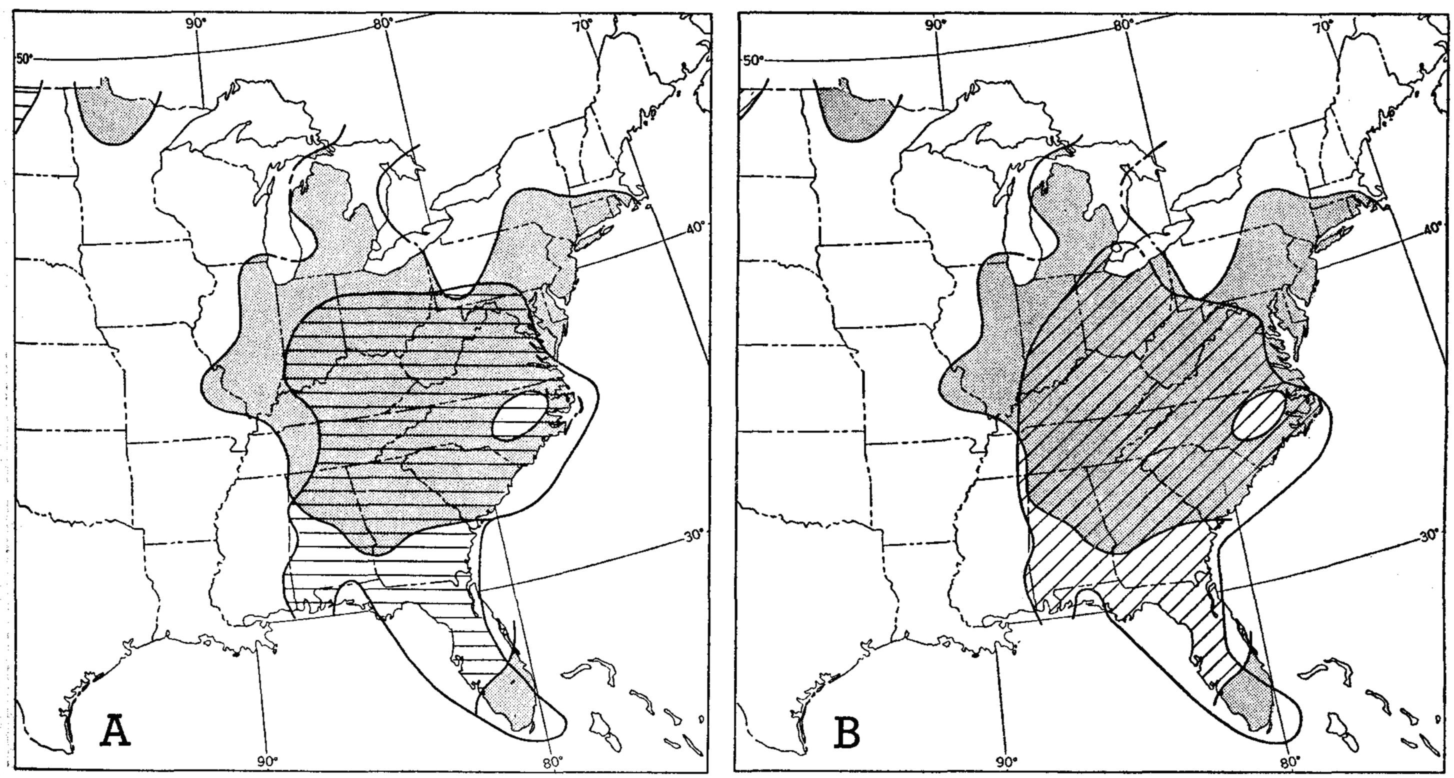

FIGURE 4.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 1500 GMT, January 2–0300 GMT, January 3, 1953.

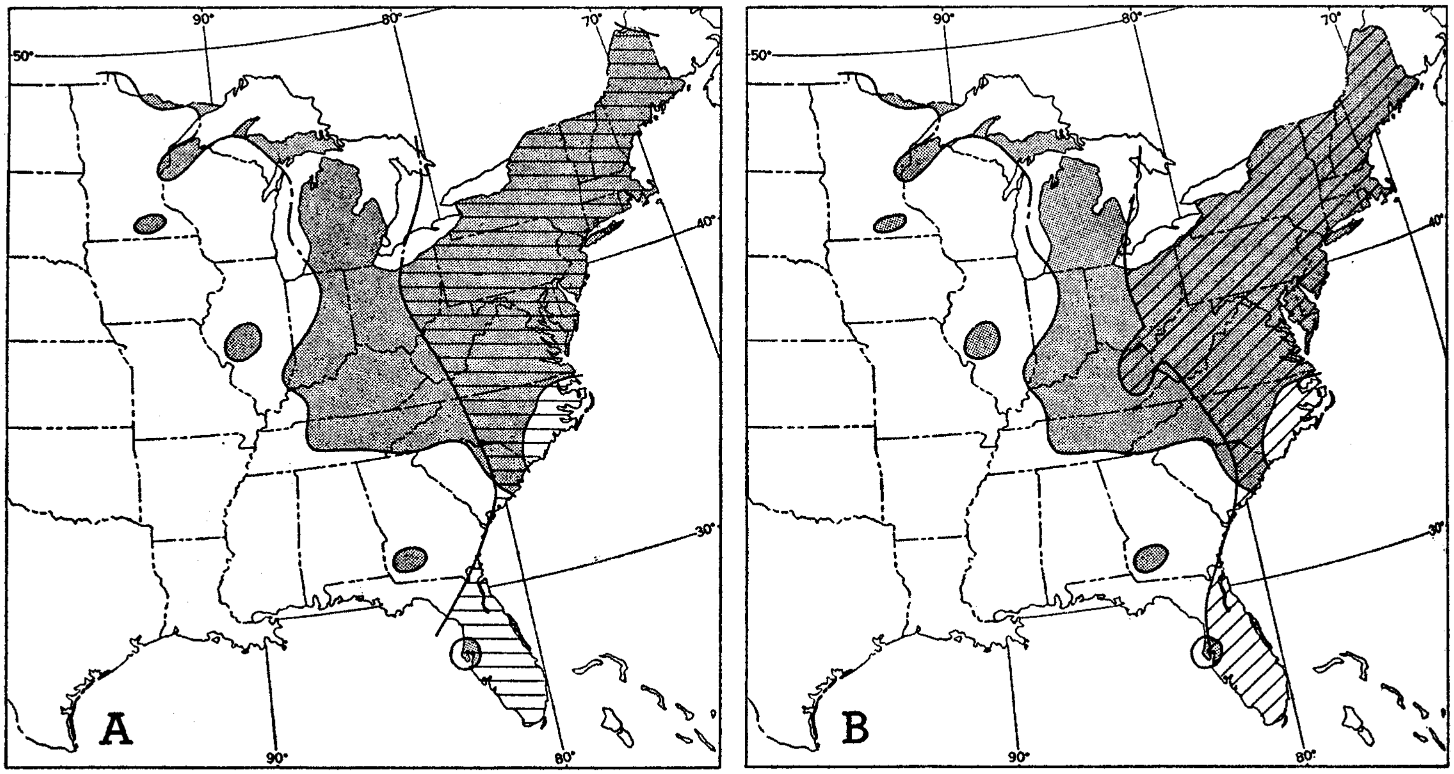

FIGURE 5.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 0300–1500 GMT, January 3, 1953.

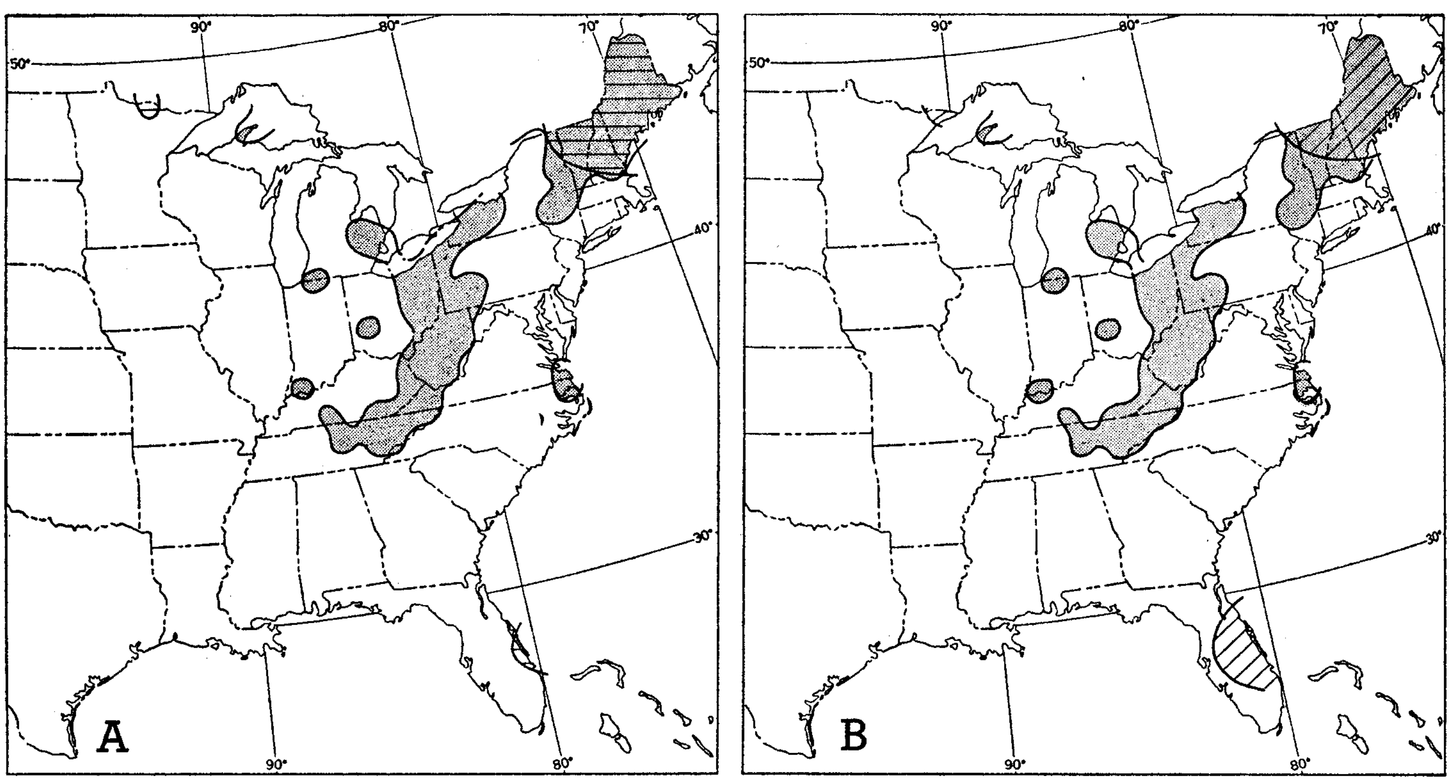

FIGURE 6.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 1500 GMT January 3–0300 GMT January 4, 1953.

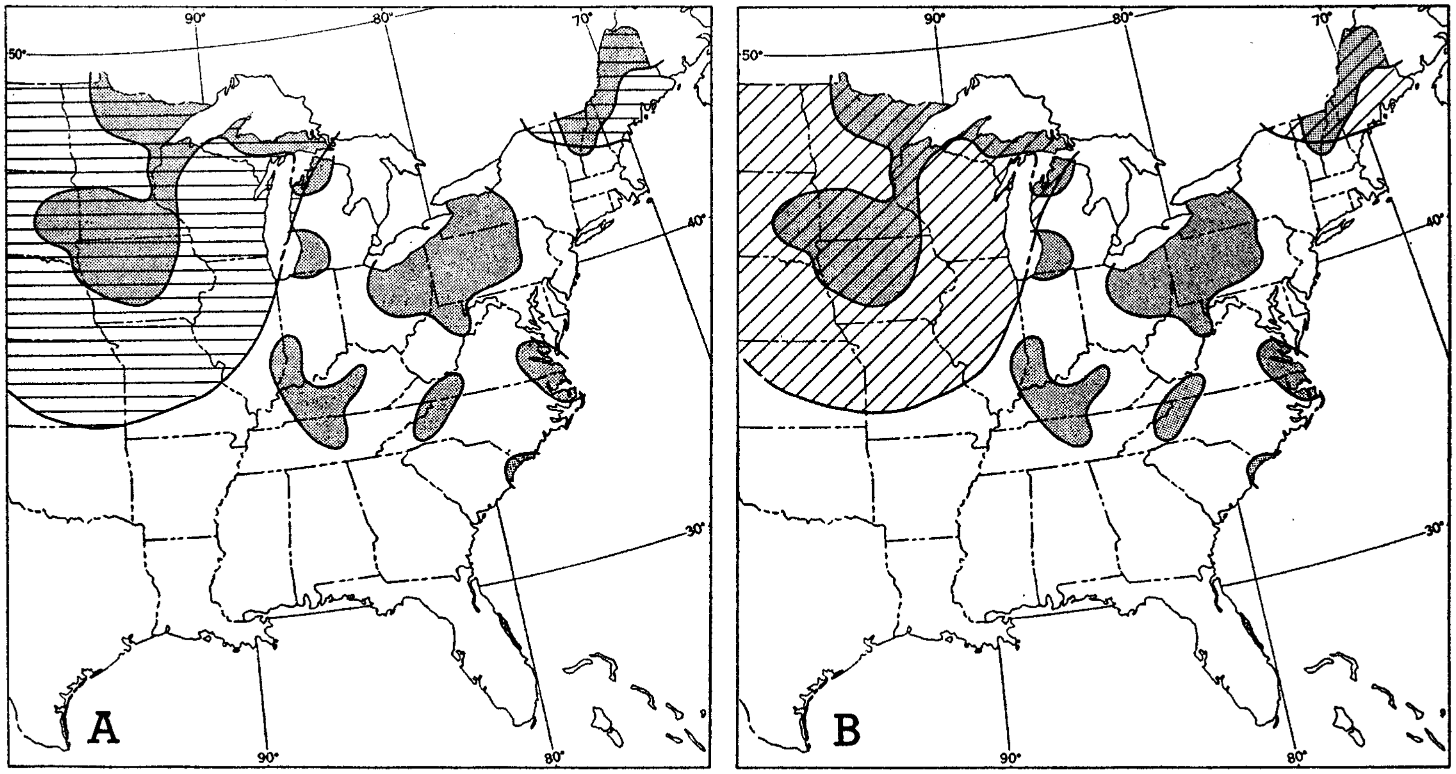

FIGURE 7.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 0300–1500 GMT, January 4, 1953.

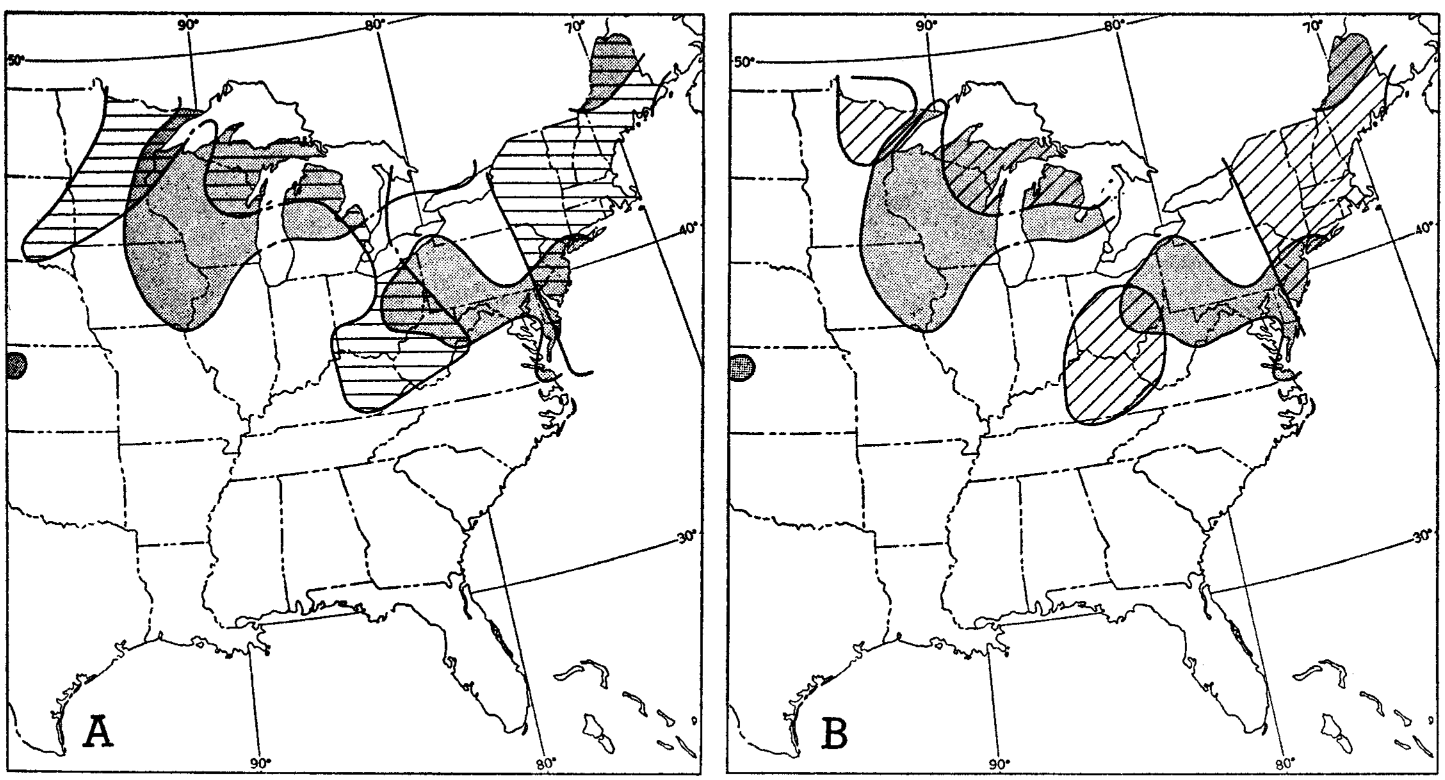

FIGURE 8.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 1500 GMT January 4–0300 GMT January 5, 1953.

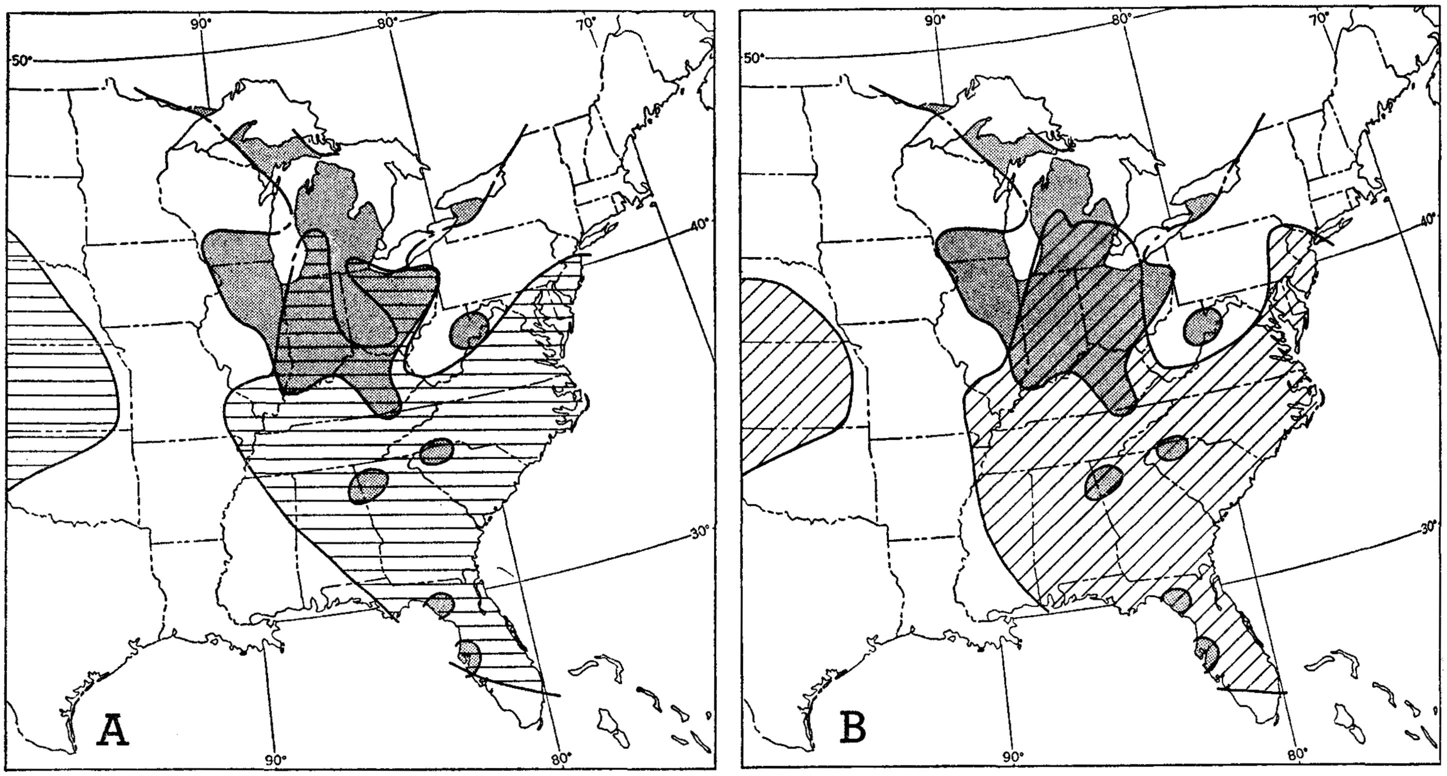

FIGURE 9.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 0300–1500 GMT, January 5, 1953.

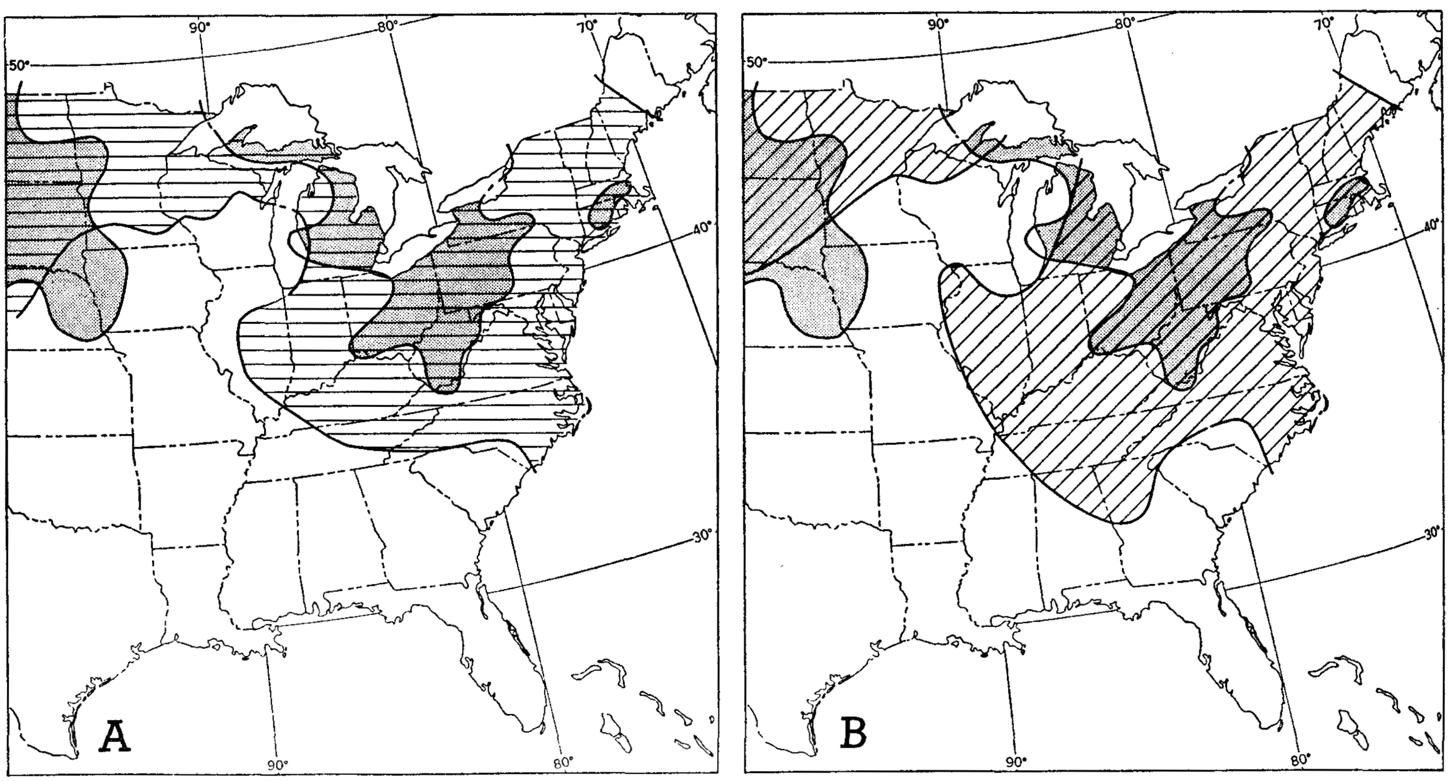

FIGURE 10.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 1500 GMT, January 5–0300 GMT, January 6, 1953.

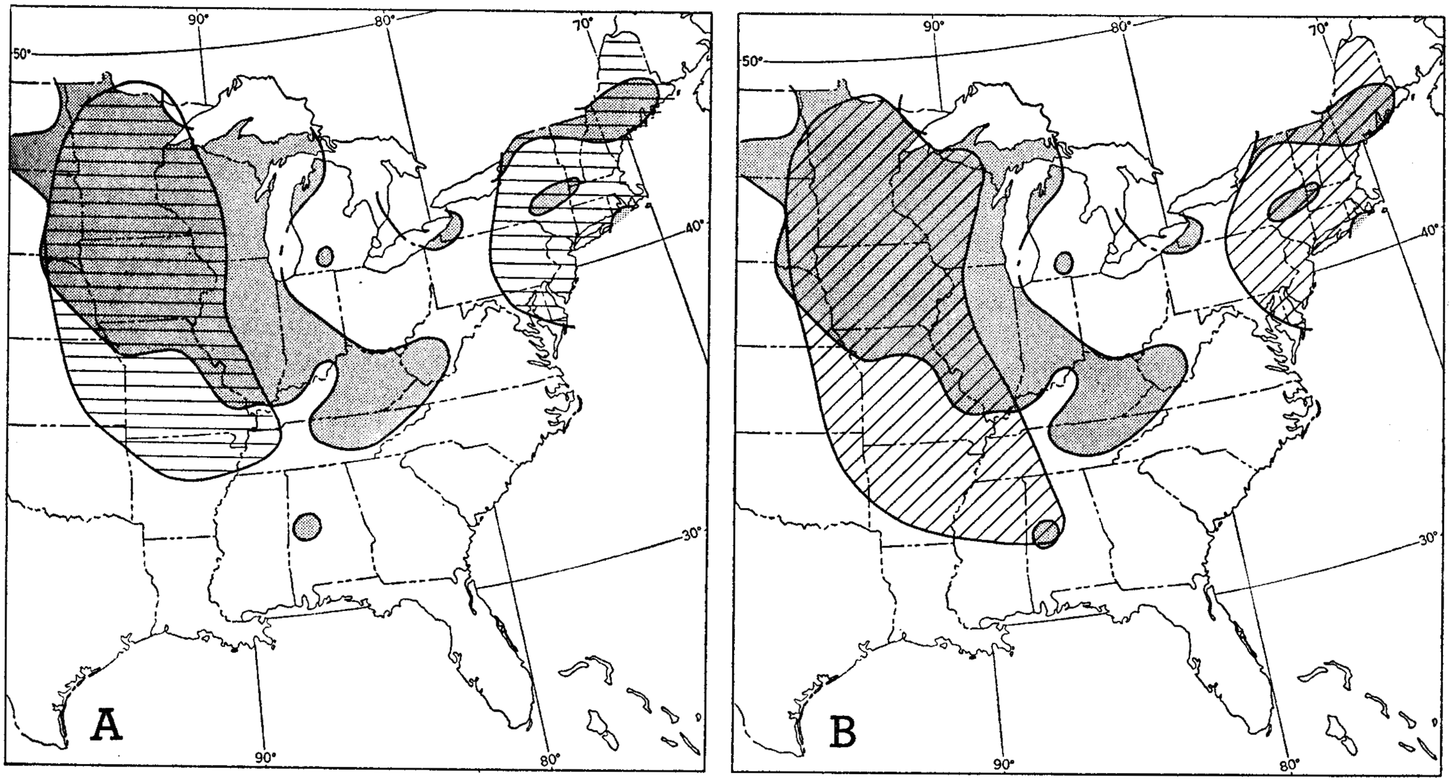

Figure 11.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 0300–1500 GMT, January 6, 1953.

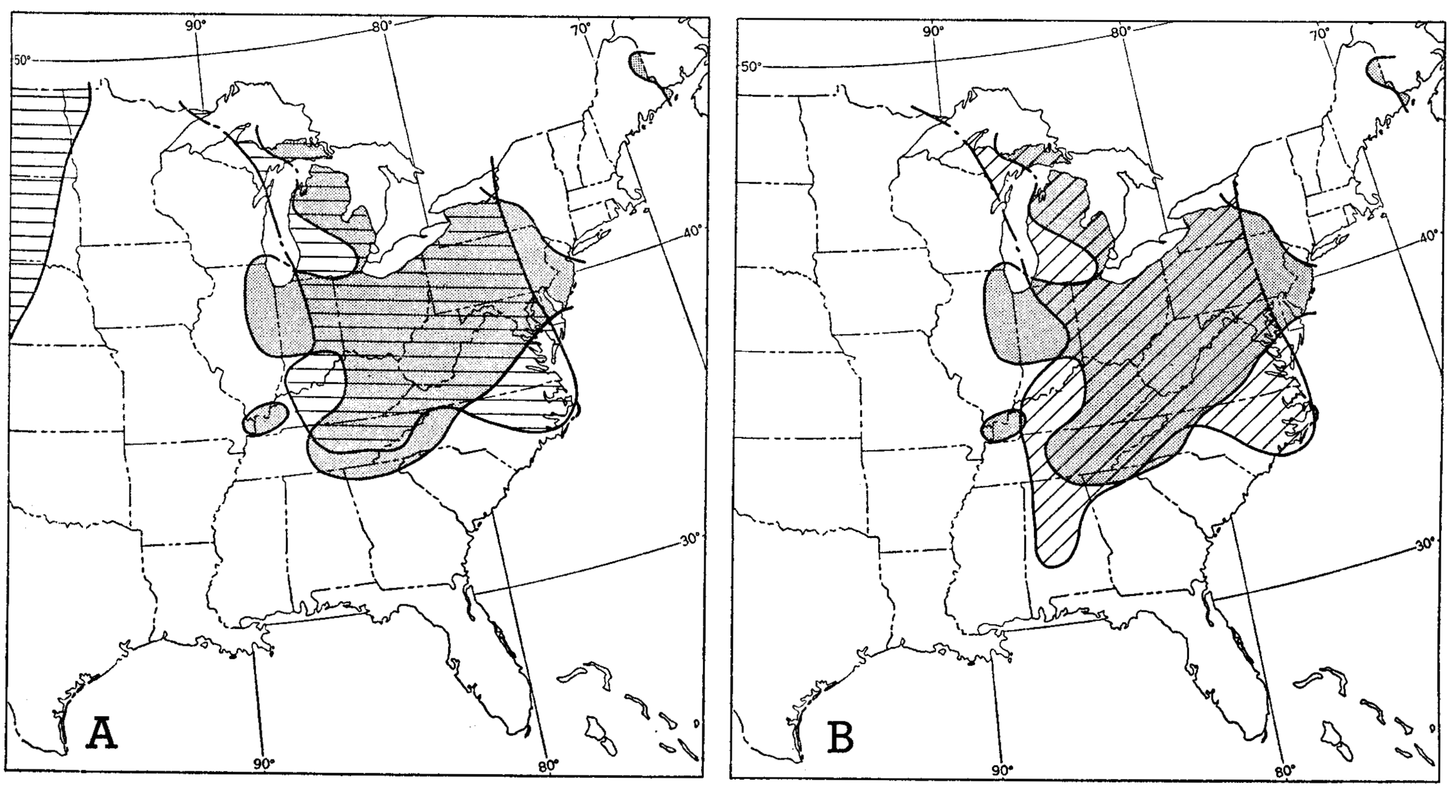

Figure 12.—Observed precipitation (shaded) superimposed on (A) computed precipitation not including topographic effects (horizontal hatching), and (B) computed precipitation including topographic effects (diagonal hatching). 1500 GMT, January 6–0300 GMT, January 7, 1953.

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tion can be evaluated graphically to give a quantitative precipitation forecast. For this experiment, however, only the boundary of precipitation ($P=0.01$ inch) was computed.

DISCUSSION OF RESULTS

The period chosen for the computations, 0300 GMT, January 2, 1953, through 0300 GMT, January 7, 1953, provided the movement through the eastern half of the United States of two broad-scale storm systems with their associated well-defined precipitation patterns.

The results of the ten computations in this period are compared qualitatively in figures 3 to 12 with the observed precipitation.

Verification on a rain, no-rain basis was made at 73 points, corresponding to selected first-order stations east of 100° W. longitude. The results are shown in table 1.

TABLE 1.—*Verification of rain, no-rain computations at 73 points, Jan. 2–7, 1953*

12 hours beginning		Without orographic lift					With orographic lift				
GMT	1953	RR	NN	RN	NR	%	RR	NN	RN	NR	%
03	Jan. 2	18	39	14	2	78	18	39	14	2	78
15	Jan. 2	14	45	6	8	81	14	46	5	8	82
03	Jan. 3	19	37	8	9	77	22	35	10	6	78
15	Jan. 3	8	55	2	8	86	8	56	1	8	88
03	Jan. 4	9	41	17	6	69	9	42	16	6	70
15	Jan. 4	9	46	13	5	75	9	49	10	5	80
03	Jan. 5	6	48	15	4	74	7	45	18	3	71
15	Jan. 5	13	38	21	1	70	13	36	23	1	67
03	Jan. 6	13	38	13	9	70	13	36	15	9	67
15	Jan. 6	13	46	8	6	81	12	48	6	7	82
Overall		122	433	117	58	76	125	432	118	55	76

Notes:
RR: Rain computed, rain observed.
NN: No rain computed, no rain observed.
RN: Rain computed, no rain observed.
NR: No rain computed, rain observed.
(Rain defined as 0.01 inch precipitation or more.)
%: Percentage correct.

During the first 48 hours and again during the last 12 hours, when well-defined systems were moving across the eastern United States, agreement between computed and observed precipitation was good. During the middle period, when there was no dominant surface storm center over the area, the method tended to overforecast badly and scores dropped. Taking into account the evaporation of the falling precipitation, which has been neglected, would tend to counteract this and improve the verification. Inclusion of topographic effects improved the method slightly, particularly during those periods when a well-defined storm center was the dominant feature. During such periods surface winds are stronger, producing larger vertical velocities at the ground which can make an appreciable contribution to precipitation.

Results were slightly but not significantly better than those of [2], which used the horizontal divergence of observed winds, smoothed horizontally, to compute vertical velocity.

CONCLUSIONS

The degree of correspondence between the computed and the observed precipitation areas suggests that vertical velocities can be deduced from vorticity changes by this graphical technique with considerable skill. This technique is too laborious for use in day-to-day forecasting, particularly when one considers that the baroclinic models (e. g. [7]) used in numerical weather prediction yield vertical motion as one of the prognostic elements.

An improvement in results might be expected if the time interval over which the vorticity changes are computed were reduced to one or at most a few hours rather than 12 hours. This is borne out by the fact that when the pressure systems, and thus the vorticity patterns, were well-defined and large in extent, so that an average of the values at the beginning and end of the time period fairly well approximated the true mean value, the computations were fairly good.

This study gives a preliminary indication of the extent to which the vertical motion associated with large-scale circulations can explain precipitation patterns. However, observed precipitation patterns are of smaller scale than those computed, resulting from the superposition upon the large-scale flow of smaller-scale features, such as convection cells. These small-scale processes should now be studied and it is hoped that analysis of the errors in these ten cases will yield information on the relationships between the large-scale flow and the small-scale phenomena.

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